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"PEAK-AVERAGE" AND "AVERAGE-PEAK" RADAR CROSS SECTIONS OF WAKES (OR CHAFF)

TECHNICAL MEMORANDUM TM-6/331

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ABSTRACT

Expressions are given comparing the "peak-average" to "average-peak" radar cross section for random distributed scatterers such as re-entry wakes or chaff. In order to relate the peak-average (which is a conventional measure) to the average-peak (which is more-simply determined), the form and extent of the scatterer must be known. Ratios of these quantities are computed for three different forms of "average pulse shape": uniform, triangular, and exponential.

AUTHORIZATION

The research described in this report was performed at the Electronics Research Laboratories of Columbia University. This report was prepared by I. Weissman.

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"PEAK-AVERAGE" AND "AVERAGE-PEAK" RADAR CROSS SECTIONS OF WAKES (OR CHAFF)

One frequently encounters the expression "peak cross section" in reference to radar echoes from random, distributed reflectors such as re-entry wakes or chaff. This is customarily taken to mean the peak of the average of a group of received pulse shapes; that is, the echo powers corresponding to a particular range-delay are averaged over a number of successive pulses, the process is repeated for different range-delays, and the peak value of the resulting "average" pulse shape is specified.

A less direct, but much simpler procedure consists of just noting the peak of each successive radar return, wherever in range this peak may occur, and then averaging these peak received powers over a number of returns. The "average-peak" radar cross section (RCS) is obtained by this method. This procedure might be employed to circumvent, for example, laborious reading of A-scope films of wake echoes.

It is clear that the measured average-peak RCS, $\overline{\sigma_{max}}$, will always exceed the peak-average RCS, $(\bar{\sigma})_{max}$, by an amount which depends on the length and the exact form of the extended pulse return. In particular, we define the ratio K, of the expected-peak RCS to the peak-expected RCS:

$$K = \frac{E(\sigma_{max})}{[E(\sigma)]_{max}} \tag{1}$$

This ratio K will tell us by how much, for a sufficiently large number of echoes, the average-peak RCS will exceed the peak-average.

Figure 1(a) illustrates individual successive received pulses from a randomly distributed scatterer, with the echo powers represented in terms of RCS vs range, i.e., σ vs R. The peaks for each return are indicated by σ_{max}. Figure 1(b) shows the average pulse shape as estimated from a large number of independent, statistically similar returns; the peak of this characteristic is indicated by $[E(\sigma)]_{max}$.

In order to find $E(\sigma_{max})$, as estimated from a large number of statistically similar returns, we must first find the probability density of σ_{max}.

It is convenient, at this point, to represent each return by n <u>independent</u> range sample values, as shown. This can always be done, without loss of generality, by properly choosing n. Now, to say that, for a given pulse, σ_{max} is less than some value, σ, is to say that the samples $\sigma(1)$, $\sigma(2) \ldots \sigma(n)$ for that pulse are <u>all</u> less than σ. Therefore, the cumulative distribution function for σ_{max} is

$$F_{\sigma_{max}}(\sigma) = Pr\left\{\sigma_{max} < \sigma\right\} = Pr\left\{\sigma(1) < \sigma, \sigma(2) < \sigma, \ldots \right.$$

$$\left. \ldots \sigma(n) < \sigma\right\} = \prod_{i=1}^{n} Pr\left\{\sigma(i) < \sigma\right\} = \prod_{i=1}^{n} F_{\sigma_i}(\sigma) \tag{2}$$

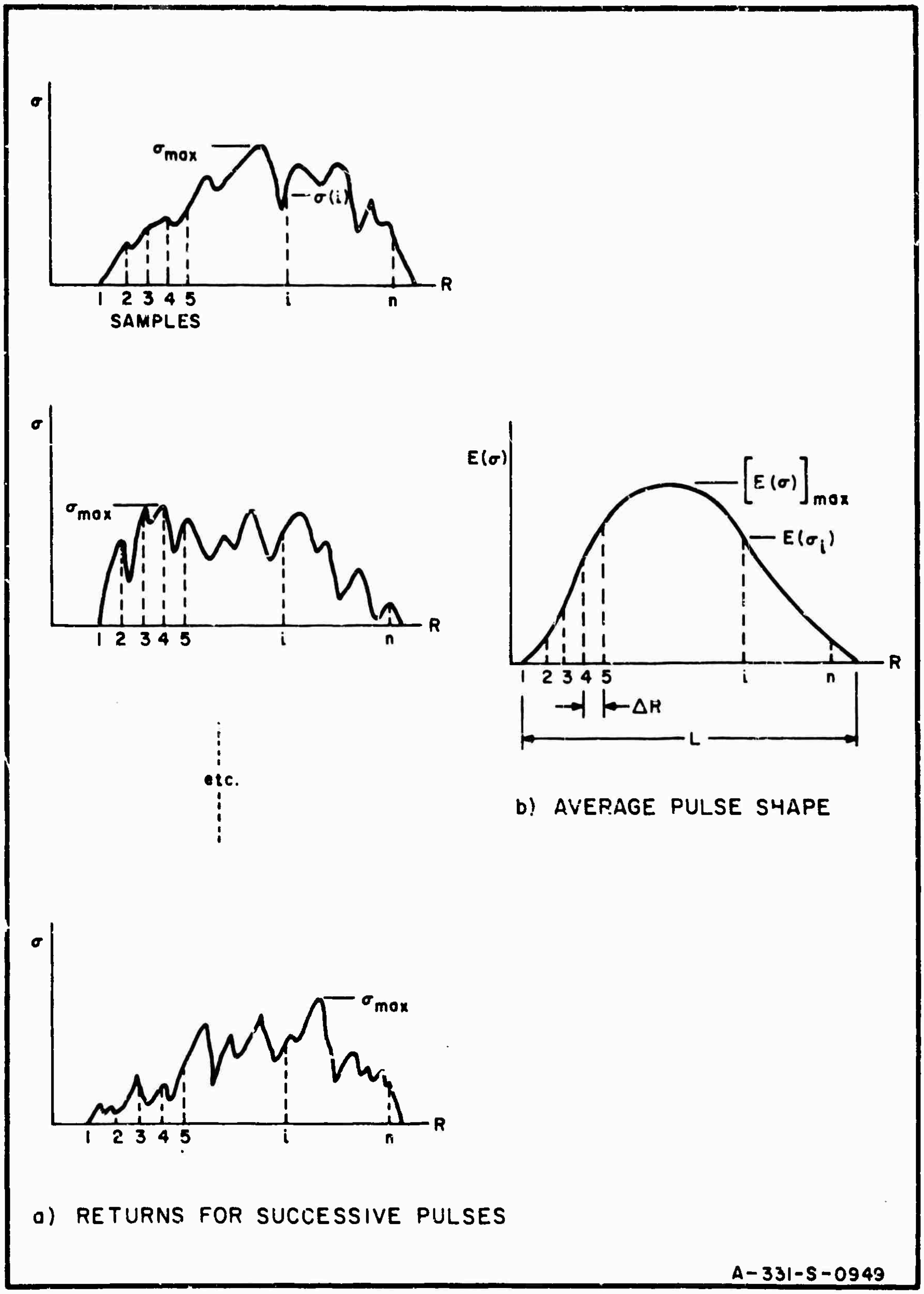

FIG. 1 RADAR ECHOES FROM RANDOM DISTRIBUTED SCATTERER

since the individual range samples were taken to be statistically independent. The expected-peak RCS can then be found:

$$E(\sigma_{max}) = \int_0^\infty \sigma \cdot f_{\sigma_{max}}(\sigma) \cdot d\sigma \tag{3}$$

where the density function is $f_{\sigma_{max}}(\sigma) = F'_{\sigma_{max}}(\sigma)$.

If it is assumed that each range sample represents echo contributions from many similar independent scattering sources (i.e., a large number of "turbs" or chaff elements within a range resolution "cell"), the RCS tends to have an exponential probability density. (The echo signal amplitude has a Rayleigh distribution). Under this assumption, which should be valid for many cases of interest,

$$F_{\sigma_{max}}(\sigma) = \prod_{i=1}^{n} \left[1 - \varepsilon^{-\sigma/E(\sigma_i)}\right] \quad ; \quad \sigma \geq 0 \tag{4}$$

where $E(\sigma_i)$ is the expected RCS for the i^{th} sample.

The exact number of independent range samples, n, used to represent the return from the distributed target depends on the spatial extent of the scatterer and also upon the transmitted radar spectrum (i.e., the autocorrelation function of the echo power). When the characteristic correlation length along the scatterer (turb size) is sufficiently less than the radar range resolution, the number of independent range samples is of the order of

$$n \approx \frac{\text{Range Extent of Scatterer}}{\text{Range Resolution}}$$

$$= \frac{L}{\Delta} . \tag{5}$$

where $L > \Delta R$.

Figure 2 illustrates three forms of average pulse shapes for which K has been computed. For the "uniform" pulse shape (Fig. 2a)

$$E(\sigma_i) = [E(\sigma)]_{max} = \text{constant}$$

and

$$F_{\sigma_{max}}(\sigma) = \left\{1 - \epsilon^{-\sigma/[E(\sigma)]_{max}}\right\}^n \tag{6}$$

from which K is calculated (using Eqs. 1 and 3) and plotted as a function of n in Fig. 3.

For the "triangular" echo shape (Fig. 2b),

$$E(\sigma_i) = \frac{(n - i + 1)}{n} \cdot [E(\sigma)]_{max}$$

and

$$F_{\sigma_{max}}(\sigma) = \prod_{i=1}^{n} \left\{1 - \epsilon^{-\frac{n}{(n-i+1)} \cdot \frac{\sigma}{[E(\sigma)]_{max}}}\right\} \tag{7}$$

for which the resulting K values are also shown in Fig. 3.

As a third example, for the "exponential" pulse shape (Fig. 2c) where $L = n \cdot \Delta R$ is the range extent at which the

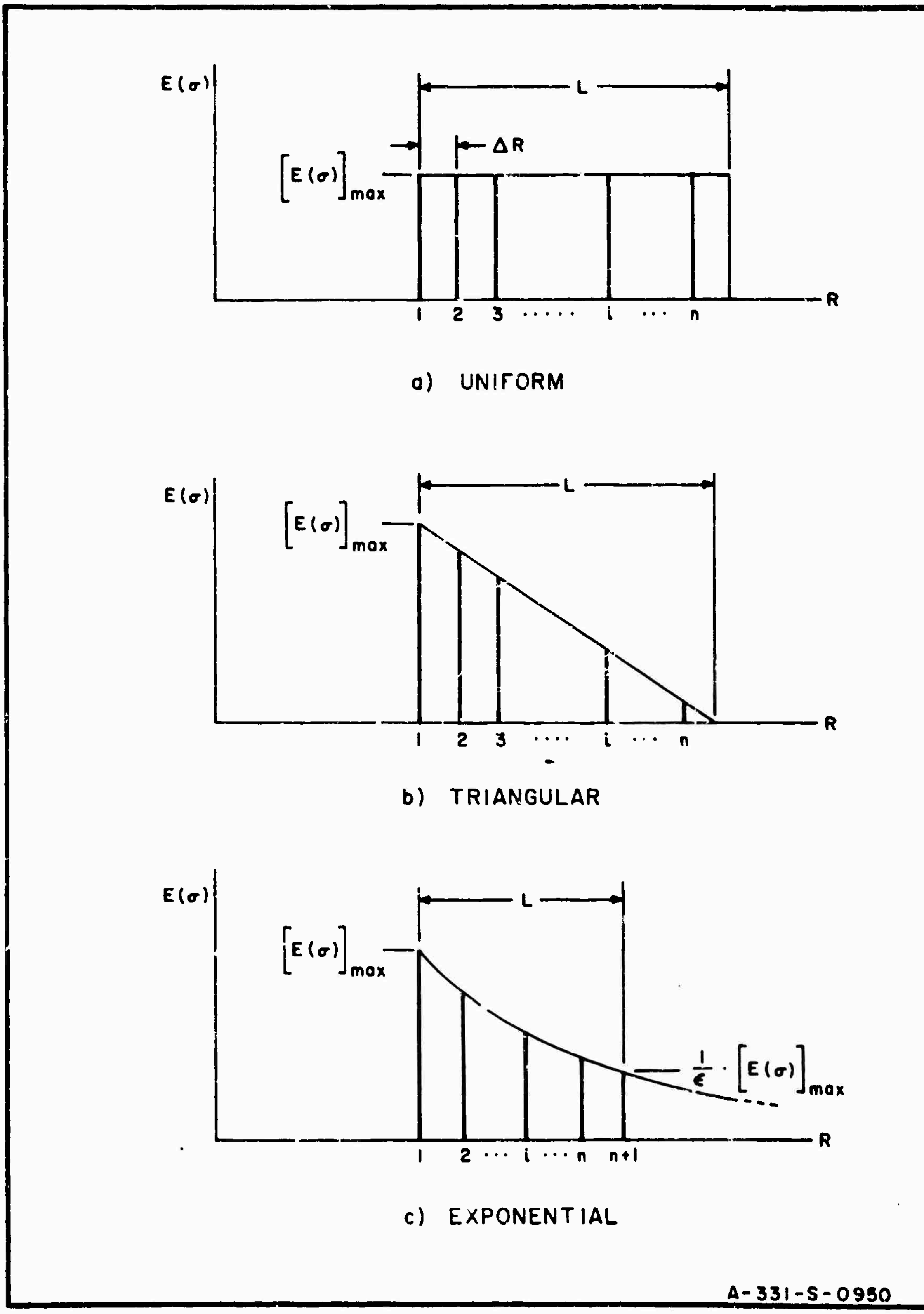

FIG. 2 VARIOUS AVERAGE ECHO PULSE SHAPES

pulse decays to $1/\epsilon$ of its peak value

$$E(\sigma_i) = [E(\sigma)]_{max} \cdot \epsilon^{-\frac{i-1}{n}} \tag{8}$$

and

$$F_{\sigma_{max}}(\sigma) = \prod_{i=1}^{\infty} \left\{ 1 - \epsilon^{-\sigma \cdot \left[\exp\left(\frac{i-1}{n}\right)\right] / [E(\sigma)]_{max}} \right\} \tag{9}$$

for which K is also plotted in Fig. 3.

The suggestions of Dr. Lee Abramson are acknowledged and appreciated.

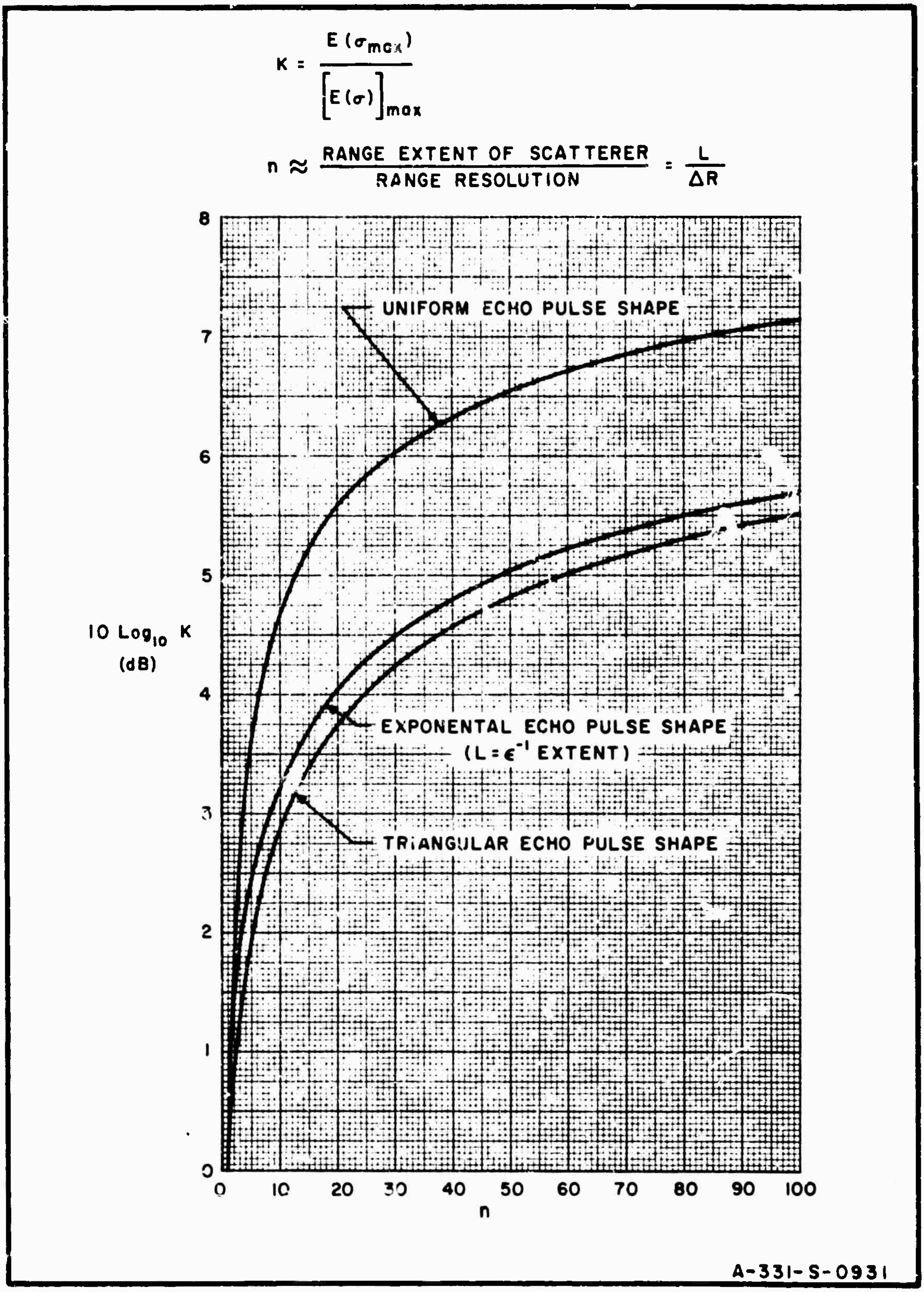

FIG. 3 RATIO OF EXPECTED-PEAK TO PEAK-EXPECTED RCS OF RANDOM DISTRIBUTED SCATTERER

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